

LABORATORY VI MECHANICAL OSCILLATIONS

In most of the laboratory problems so far objects have been moving with constant acceleration because the total force acting on that object was constant. In this set of laboratory problems the force on an object, and thus its acceleration, changes with position.

You are familiar with many objects that oscillate -- a tuning fork, the balance wheel of a mechanical watch, a pendulum, or the strings of a guitar. At the atomic level, atoms oscillate within molecules, and molecules within solids. All of these objects are subjected to forces which change with position. Springs are a common example of objects that exert this type of force.

In this lab you will study oscillatory motion caused by springs exerting a force on an object. You will use different methods to determine the strength of the force exerted by different spring configurations, and investigate what quantities determine the oscillation frequency of systems.

OBJECTIVES:

After successfully completing this laboratory, you should be able to:

- provide a qualitative explanation of the behavior of oscillating systems using the concepts of restoring force and equilibrium position.
- identify the physical quantities which influence the period (or frequency) of the oscillatory motion and describe this influence quantitatively
- demonstrate a working knowledge of the mathematical description of an oscillator's motion.
- describe qualitatively the effect of additional forces on an oscillator's motion.

LABORATORY VI – MECHANICAL OSCILLATIONS

PREPARATION:

Young & Freedman, University Physics, 11th ed. Ch. 13.

Before coming to lab you should be able to:

- Describe the similarities and differences in the behavior of the sine and cosine functions.
- Recognized the difference between amplitude, frequency, angular frequency, and period for repetitive motion.
- Determine the force on an object exerted by a spring using the concept of a spring constant.

PROBLEM #1: MEASURING SPRING CONSTANTS

You are selecting springs for a large antique clock, and to determine the force they exert, you need to know their spring constants. One book recommends a static approach: hang objects of different weights on the spring and measure the displacement from equilibrium. Another book suggests a dynamic approach: hang an object on the end of a spring and measure its oscillation frequency. To determine if these two approaches give you the same quantity, you calculate the "spring constant" from both sets of measurements, make the measurements, and compare.



Does measuring the oscillation frequency give a different spring constant than measuring a displacement from equilibrium?

EQUIPMENT

You can hang a spring from a metal rod that is fastened on a table clamp. Then you can attach an object to the hanging end of the spring. You can vary the mass of the object. A meter stick, a triple-beam balance, a stopwatch, a mass hanger, a mass set and the video analysis equipment are available for this experiment.



PREDICTIONS

1. Calculate the spring constant from the displacement of an object hung on the spring. How do you check this prediction?
2. Calculate the spring constant from the period of oscillation of an object hung on the spring. How do you check this prediction?

METHOD QUESTIONS

To complete your predictions, it is useful to apply a problem-solving strategy such as the one outlined below:

Method #1: Suppose you hang objects of several different masses on a spring and measure the vertical displacement of each object.

1. Make two sketches of the situation, one before you attach a mass to a spring, and one after a mass is suspended from the spring and is at rest. Draw a coordinate system and label the position where the spring is unstretched, the stretched position, the mass of the object, and the spring constant. Assume the springs are massless.

2. Draw a force diagram of a hanging object at rest. Label the forces. Newton's second law gives the equation of motion for the hanging object. Solve this equation for the spring constant.

3. Use your equation to sketch the expected shape of the graph of displacement (from the unstretched position) versus weight for the object-spring system. How is the slope of this graph related to the spring constant?

Method #2: Suppose you hang an object from the spring, start it oscillating, and measure the *period* of oscillation.

1. Make a sketch of the oscillating system at a time when the object is *below* its equilibrium position. Draw this sketch to the side of the two sketches drawn for method #1. Identify and label this new position on the same coordinate axis.

2. Draw an force diagram of the object at this new position. Label the forces.

3. Apply Newton's second law as the equation of motion for the object at each of the above positions.

When the object is below its equilibrium position, how is the stretch of the spring from its unstretched position related to the position of the system's (spring & object) equilibrium position and its additional displacement from that equilibrium position to its new position.

4. Solve your equations for acceleration of the object as a function of the mass of the suspended object, the spring constant, and the displacement of the spring/object system from its equilibrium position.

5. Try a periodic solution (sin or cos) to your equation of motion (Newton's second law). For what period of the system will your solution to the equation of motion work? How is the frequency of the system related to its period of oscillation?

EXPLORATION

Method #1: Select a series of masses that give a usable range of displacements. The smallest mass must be much greater than the mass of the spring to fulfill the massless spring assumption. **The largest mass should not pull the spring past its elastic limit (about 60 cm). Beyond that point you will damage the spring.** Decide on a procedure that allows you to measure the displacement of the spring-object system in a consistent manner. Decide how many measurements you will need to make a reliable determination of the spring constant.

Method #2: Secure one end of the spring safely to the metal rod and select a mass that gives a regular oscillation without excessive wobbling to the hanging end of the spring. Again, the largest mass should not pull the spring past its elastic limit and the smallest mass should be much greater than the mass of the spring. Practice starting the mass in vertical motion smoothly and consistently.

Practice making a video to record the motion of the spring-object system. Decide how to measure the period of oscillation of the spring-object system by video and stopwatch. How can you minimize the uncertainty introduced by your reaction time in starting and stopping the stopwatch? How many times should you measure the period to get a reliable value? How will you determine the uncertainty in the period?

MEASUREMENT

Method #1: Record the masses of different hanging objects and the corresponding displacements.

Method #2: For each hanging object, record the mass of the object. Use a stopwatch to roughly determine the period of the oscillation and then make a video of the motion of the hanging object. Repeat the same procedure for objects with different masses.

Analyze your data as you go along so you can decide how many measurements you need to make to determine the spring constant accurately and reliably.

ANALYSIS

Method #1: Make a graph of displacement versus weight for the object-spring system. From the slope of this graph, calculate the value of the spring constant, including the uncertainty.

Method #2: Digitize your videos to determine the period of each oscillation. You may use the period by stopwatch as a predicted parameter in your fit equations. Make a graph of period (or frequency) versus mass for the object-spring system. If this graph is not a straight line, make another graph with the period as a function of mass to a power that is a straight line. From the slope of the straight-line graph, calculate the value of the spring constant, including the uncertainty.

CONCLUSION

How do the two values of the spring constant compare? Which method is faster? Which method do you feel is the most reliable? Justify your answers.

PROBLEM #2: THE EFFECTIVE SPRING CONSTANT

Your company has bought the prototype for a new flow regulator from a local inventor. Your job is to prepare the prototype for mass production. While studying the prototype, you notice the inventor used some rather innovative spring configurations to supply the tension needed for the regulator valve. In one location the inventor had fastened two different springs side-by-side, as in Figure A below. In another location the inventor attached two different springs end-to-end, as in Figure B below.

To decrease the cost and increase the reliability of the flow regulator for mass production, you need to replace each spring configuration with a single spring. These replacement springs must exert the same forces when stretched the same amount as the original spring configurations.



What are the spring constants of two single springs that would replace the side-by-side and end-to-end spring configurations?

The spring constant for a single spring that replaces a configuration of springs is called its *effective spring constant*.

EQUIPMENT

You have two different springs that have the same unstretched length, but different spring constants k_1 and k_2 . These springs can be hung vertically side-by-side (Figure A) or end-to-end (Figure B). You will also have a meter stick, a stopwatch, a metal rod, a wooden rod, a table clamp, a mass hanger, a mass set and the video analysis equipment.

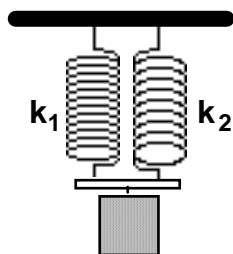


Figure A

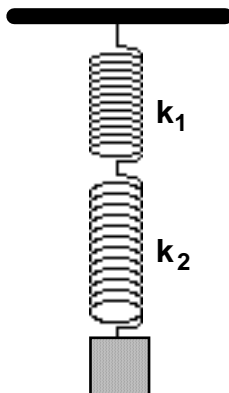


Figure B

PREDICTIONS

1. Calculate the effective spring constant for the side-by-side configuration (Figure A) in terms of the two spring constants k_1 and k_2 .
2. Calculate the effective spring constant for the end-to-end configuration (Figure B) in terms of the two spring constants k_1 and k_2 .
3. *Is the effective spring constant larger when the two springs are connected side-by-side or end-to-end? Explain.*

METHOD QUESTIONS

To complete your predictions, it is useful to apply a problem-solving strategy such as the one outlined below. *Apply the strategy first to the side-by-side configuration, then repeat for the end-to-end configuration:*

1. Make a sketch of the spring configuration similar to one of the drawings in the Equipment section (Figure A or Figure B). Draw a coordinate system and label the positions of each unstretched spring, the final stretched position of each spring, the two spring constants, and the mass of the object suspended. Assume that the springs are massless.

For the side-by-side configuration, assume that the light bar attached to the springs remains horizontal (i.e. it does not twist).

Now make a second sketch of a single (massless) spring with spring constant k' that has the same object suspended from it and the same total stretch as the combined springs. Be sure to label this second sketch.

2. Draw force diagrams of the object suspended from the combined springs and the same object suspended from the single replacement spring. Label the forces.

For the end-to-end configuration, draw an additional force diagram of a point at the connection of the two springs.

3. Apply Newton's laws as the equations of motion for the object suspended from the combined springs and the object suspended from the single replacement spring.

How is the total stretch of the combined springs related to the stretch of each of the springs?

For the end-to-end configuration: At the connection point of the two springs, what is the force of the top spring on the bottom spring? What is the force of the bottom spring on the top spring?

4. Solve your equations for the effective spring constant (k') for the single replacement spring in terms of the two spring constants.

EXPLORATION

To test your predictions, you must decide how to measure each spring constant of the two springs and the effective spring constants of the side-by-side and end-to-end configurations.

From your results of Problem #1, select the best method for measuring spring constants. Justify your choice. **DO NOT STRETCH THE SPRINGS PAST THEIR ELASTIC LIMIT (ABOUT 60 CM) OR YOU WILL DAMAGE THEM.**

Perform an exploration consistent with your selected method. If necessary, refer back to the appropriate Exploration section of Problem #1.

Remember, the smallest mass must be much greater than the mass of the spring to fulfill the massless spring assumption. The largest mass should not pull the spring past its elastic limit.

Outline your measurement plan.

MEASUREMENT

Make the measurements that are consistent with your selected method. If necessary, refer back to the appropriate Measurement section of Problem #1. What are the uncertainties in your measurements?

ANALYSIS

Determine the effective spring constants (with uncertainties) of the side-by-side spring configuration and the end-to-end spring configuration. If necessary, refer back to Problem #1 for the analysis technique consistent with your selected method.

Determine the spring constants of the two springs. Calculate the effective spring constants (with uncertainties) of the two configurations using your Prediction equations.

PROBLEM #2: EFFECTIVE SPRING CONSTANT

How do the measured and predicted values of the effective spring constants for the two configurations compare?

CONCLUSION

What are the effective spring constants of a side-by-side spring configuration and an end-to-end spring configuration? Which is larger? Did your measured values agree with your initial predictions? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

PROBLEM #3: OSCILLATION FREQUENCY WITH TWO SPRINGS

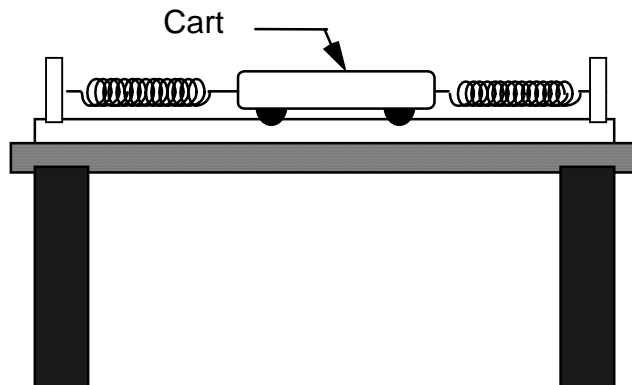
You have a summer job with a research group at the University. Because of your physics background, your supervisor asks you to design equipment to measure earthquake aftershocks. The calibration sensor needs to be isolated from the earth movements, yet free to move. You decide to place the sensor on a track cart and attach a spring to both sides of the cart. You should now be able to measure the component of the aftershocks along the axis defined by the track. To make any quantitative measurements with the sensor you need to know the frequency of oscillation for the cart as a function of the spring constants and the mass of the cart.



What is the frequency of the oscillating cart?

EQUIPMENT

You will have an aluminum track, two adjustable end stops, two oscillation springs, a meter stick, a stopwatch, a cart and the video analysis equipment.



PREDICTION

Calculate the frequency of the cart in terms of its mass and the two spring constants.

METHOD QUESTIONS

To complete your prediction, it is useful to use a problem-solving strategy such as the one outlined below:

1. Make two sketches of the oscillating cart, one at its equilibrium position, and one at some other position and time while it is oscillating. On your sketches, show the direction of the velocity and acceleration of the cart. Identify and label the known (measurable) and unknown quantities.

2. Draw a force diagram of the oscillating cart away from its equilibrium position. Label the forces.

3. Apply Newton's laws as the equation of motion for the cart.

4. Solve your equation for the acceleration, simplifying the equation until it is similar to equation 14-2.

5. Try a periodic solution (sin or cos) to your equation of motion. For what period of the system will your solution to the equation of motion work? How is the frequency of the system related to its period of oscillation? Calculate frequency of the system as a function of the mass of the cart and the two spring constants.

EXPLORATION

Decide the best method to determine the spring constants based on your results of Problem #1. **DO NOT STRETCH THE SPRINGS PAST THEIR ELASTIC LIMIT (ABOUT 60 CM) OR YOU WILL DAMAGE THEM.**

Find the best place for the adjustable end stop on the track. *Do not stretch the springs past 60 cm*, but stretch them enough so they oscillate the cart smoothly.

Practice releasing the cart smoothly. You may notice the amplitude of oscillation decreases. What's the reason for it? Does this affect the period of oscillation?

MEASUREMENT

Determine the spring constants. Record these values. What is the uncertainty in these measurements?

Record the mass of the cart. Use a stopwatch to roughly determine the period of oscillation and then make a video of the motion of the oscillating cart. You should record at least 3 cycles.

ANALYSIS

Analyze your video to find the period of oscillation. Calculate the frequency (with uncertainty) of the oscillations from your measured period.

Calculate the frequency (with uncertainty) using your Prediction equation.

How does your measured frequency compare with your predicted frequency?

CONCLUSION

What is the frequency of the oscillating cart? Did your measured frequency agree with your predicted frequency? Why or why not? What are the limitations on the accuracy of your measurements and analysis?

If you completed Problem #2: What is the effective spring constant of this configuration? How does it compare with the effective spring constants of the side-by-side and end-to-end configurations?

PROBLEM #4: OSCILLATION FREQUENCY OF AN EXTENDED SYSTEM

You are the technical advisor for the next Bruce Willis action adventure movie, *Die Even Harder*, which is filmed in Minnesota. The script calls for a spectacular stunt. Bruce Willis is dangling over a cliff from a long rope that is tied to the Bad Guy, who is on the ice-covered ledge of the cliff. The Bad Guy's elastic parachute line is tangled in a tree located several feet from the edge of the cliff. Bruce and the Bad Guy are in simple harmonic motion, and at the top of his motion, Bruce unsuccessfully tries to grab for the safety of the cliff edge while the Bad Guy reaches for his discarded knife. The script calls for Bad Guy to cut the rope just as Bruce reaches the top of his motion again.

The problem is that it is expensive to have Bruce hanging from the rope while the crew films close-ups of the Bad Guy. However, the stunt double weighs more than Bruce. The director wants to know if the stunt double will have a different motion than Bruce.



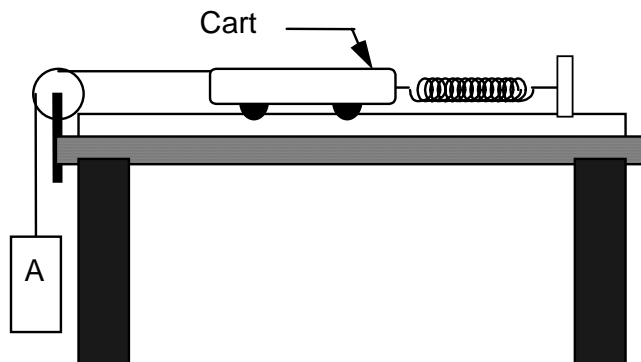
Does the frequency of oscillation depend on the mass of the hanging object?

You decide to solve this problem by modeling the situation with the equipment described below. Filming stops while you are working on this problem. If you are wrong, you are sure to lose your job.

EQUIPMENT

You have an aluminum track with an adjustable end stop, a pulley, a pulley clamp, an oscillation spring, a cart, some strings, a mass hanger, a mass set, a meter stick, a stopwatch and the video analysis equipment.

The track represents the ice-covered ledge of the cliff, the adjustable end-stop represents the tree, the spring represents the elastic cord, the cart represents the Bad Guy, the string represents the rope and the hanging object represents Bruce Willis or his stunt double in this problem.



PREDICTION

Calculate how the frequency of oscillation of the system depends on the mass of the object hanging over the table.

Use your equation to sketch the expected shape of a graph of the oscillation frequency versus hanging mass. *Will the frequency **increase**, **decrease** or **stay the same** as the hanging mass increases?*

METHOD QUESTIONS

To complete your prediction, it is useful to use a problem-solving strategy such as the one outlined below:

1. Make sketches of the situation when the cart and hanging object are at their equilibrium position and at some other time while the system is oscillating. On your sketches, show the direction of the acceleration of the cart and hanging object. Identify and label the known (measurable) and unknown quantities.

2. Draw separate force diagrams of the oscillating cart and hanging object. Label the forces.

3. Independently apply Newton's laws to the cart and to the hanging object.

4. Solve your equations for the acceleration, simplifying the equation until it is similar to equation 14-2.

5. Try a periodic solution (sin or cos) to your equation of motion. For what period of the system will your solution to the equation of motion work? How is the frequency of the system related to its period of oscillation? Calculate frequency of the system as a function of the mass of the cart, the mass of the hanging object, and the spring constant.

Now you can complete your prediction. Use your equation to sketch the expected shape of the graph of oscillation frequency versus the hanging object's mass.

EXPLORATION

If you do not know the spring constant of your spring, you should decide the best way to determine the spring constant based on your results of Problem #1.

Find the best place for the adjustable end stop on the track. **DO NOT STRETCH THE SPRING PAST 60 CM OR YOU WILL DAMAGE IT**, but stretch it enough so the cart and hanging mass oscillate smoothly.

Determine the best range of hanging masses to use.

Practice releasing the cart and hanging mass smoothly and consistently. You may notice the amplitude of oscillation decreases. What's the reason for it? Does this affect the period of oscillation?

MEASUREMENT

If necessary, determine the spring constant of your spring. What is the uncertainty in your measurement?

For each hanging object, record the masses of the cart and the hanging object. Use a stopwatch to roughly determine the period of oscillation and then make a video of the oscillating cart for each hanging object. You should record at least 3 cycles for each video.

Analyze your data as you go along, so you can determine the size and number of different hanging masses you need to use.

Collect enough data to convince yourself and others of your conclusion about how the oscillation frequency depends on the hanging mass.

ANALYSIS

For each hanging object, digitize the video to get the period of oscillation and then calculate the oscillation frequency (with uncertainty) from your measured period.

Graph the frequency versus the hanging object's mass. On the same graph, show your predicted relationship.

What are the limitations on the accuracy of your measurements and analysis? Over what range of values does the measured graph match the predicted graph best? Do the two

curves start to diverge from one another? If so, where? What does this tell you about the system?

CONCLUSION

Does the oscillation frequency increase, decrease or stay the same as the hanging object's mass increases? State your result in the most general terms supported by your analysis.

What will you tell the director? Do you think the motion of the actors in the stunt will change if the heavier stunt man is used instead of Bruce Willis? How much heavier than Bruce would the stunt man have to be to produce a noticeable difference in the oscillation frequency of the actors? Explain your reasoning in terms the director would understand so you can collect your paycheck.

PROBLEM #5: DRIVEN OSCILLATIONS

You are now prepared to calibrate your seismic detector (from Problem #3). You need to determine how the amplitude of the oscillations of the detector will vary with the frequency of the earthquake aftershocks. So you replace the end stop on the track with a device that moves the end of the spring back and forth, simulating the earth moving beneath the track. The device, called a mechanical oscillator, is designed so you can change its frequency of oscillation.

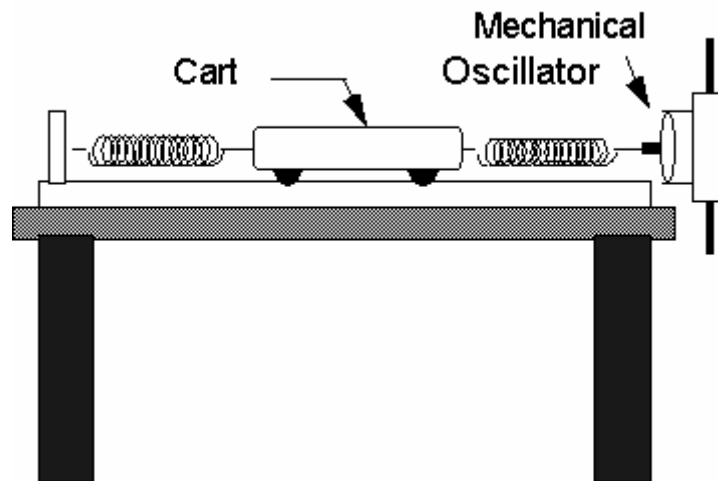


How will the amplitude of the oscillations vary with frequency?

EQUIPMENT

You will use your seismic detector apparatus and a mechanical oscillator.

The mechanical oscillator is somewhat like a loudspeaker with one end of a metal rod attached to the center, and the other end of the rod is attached to one of the springs. The oscillator is connected to a function generator which causes the rod to oscillate back and forth with adjustable frequencies which can be read from the display on the function generator.



You will also have a meter stick, a stopwatch, a metal rod, a table clamp, a stop end, two oscillation springs and two banana cables.

PREDICTION

Make your best-guess sketch of what you think a graph of the amplitude of the cart versus the frequency of the mechanical driver will look like. Assume the mechanical oscillator has a constant amplitude of a few millimeters.

EXPLORATION

Examine the mechanical oscillator. Mount it at the end of the aluminum track, using the clamp and metal rod so its shaft is aligned with the cart's motion. Connect it to the function generator, using the output marked **Lo** (for "low impedance"). Use middle or maximum amplitude to observe the oscillation of the cart at the lowest frequency possible.

Determine the accuracy of the digital display on the frequency generator by timing one of the lower frequencies.

Devise a scheme to accurately determine the amplitude of a cart on the track, and practice the technique. For each new frequency, should you restart the cart at rest?

When the mechanical oscillator is at or near the undriven frequency (natural frequency) of the cart-spring system, try to simultaneously observe the motion of the cart and the shaft of the mechanical oscillator. What is the relationship? What happens when the oscillator's frequency is twice as large as the natural frequency?

MEASUREMENT

If you do not know the natural frequency of your system when it is not driven, determine it using the technique of Problem #3.

Collect enough cart amplitude and oscillator frequency data to test your prediction. Be sure to collect several data points near the natural frequency of the system.

ANALYSIS

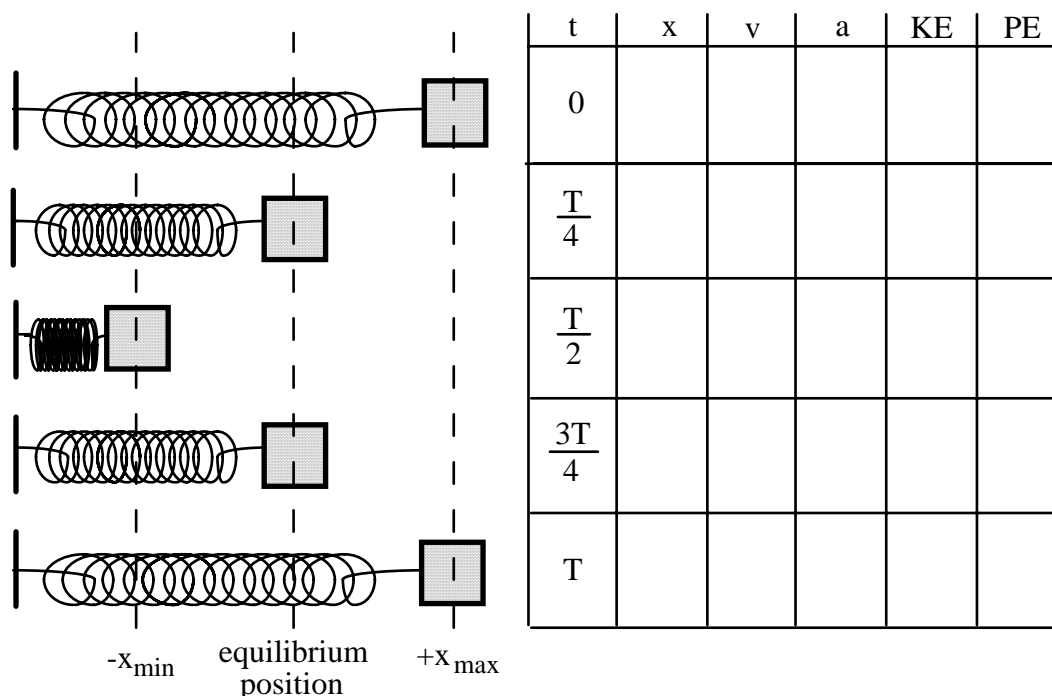
Make a graph the oscillation amplitude of the cart versus oscillator frequency. Is this the graph you had anticipated? Where is it different? Why? What is the limitation on the accuracy of your measurements and analysis?

CONCLUSION

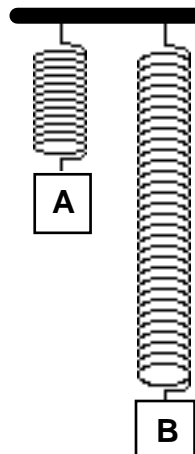
Can you explain your results? Is energy conserved? What will you tell your boss about your design for a seismic detector?

CHECK YOUR UNDERSTANDING

1. The diagram below shows an oscillating mass/spring system at times 0, $T/4$, $T/2$, $3T/4$, and T , where T is the period of oscillation. For each of these times, write an expression for the displacement (x), the velocity (v), the acceleration (a), the kinetic energy (KE), and the potential energy (PE) *in terms of the amplitude of the oscillations (A), the angular velocity (ω), and the spring constant (k).*



2. Identical masses are attached to identical springs which hang vertically. The masses are pulled down and released, but mass B is pulled further down than mass A, as shown at right.



a. Which mass will take a longer time to reach the equilibrium position? Explain.

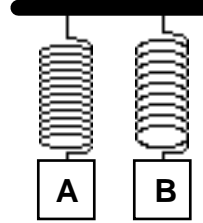
b. Which mass will have the greater acceleration at the instant of release, or will they have the same acceleration? Explain.

c. Which mass will be going faster as it passes through equilibrium, or will they have the same speed? Explain.

d. Which mass will have the greater acceleration at the equilibrium point, or will they have the same acceleration? Explain.

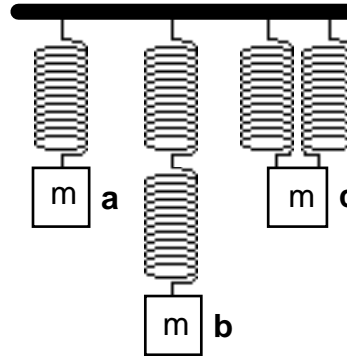
CHECK YOUR UNDERSTANDING

3. Two different masses are attached to different springs which hang vertically. Mass A is larger, but the period of simple harmonic motion is the same for both systems. They are pulled the same distance below their equilibrium positions and released from rest.



- Which spring has the greater spring constant? Explain.
- Which spring is stretched more at its equilibrium position? Explain.
- The instant after release, which mass has the greater acceleration? Explain.
- If potential energy is defined to be zero at the equilibrium position for each mass, which system has the greater total energy of motion? Explain.
- Which mass will have the greater kinetic energy as it passes through its equilibrium position? Explain.
- Which mass will have the greater speed as it passes through equilibrium? Explain.

4. Five identical springs and three identical masses are arranged as shown at right.



a. Compare the stretches of the springs at equilibrium in the three cases. Explain.

b. Which case, a, b, or c, has the greatest effective spring constant? The smallest effective spring constant? Explain.

c. Which case would execute simple harmonic motion with the greatest period? With the least period? Explain.